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Publication date:
2013

[Link to publication in Tilburg University Research Portal](#)

Citation for published version (APA):

Huisman, K. J. M., & Kort, P. M. (2013). *Strategic Capacity Investment Under uncertainty*. (CentER Discussion Paper; Vol. 2013-003). Operations research.

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No. 2013-003

**STRATEGIC CAPACITY INVESTMENT
UNDER UNCERTAINTY**

By

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10 January, 2013

ISSN 0924-7815
ISSN 2213-9532

Strategic Capacity Investment Under Uncertainty*

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December 20, 2012

Abstract

This paper considers investment decisions within an uncertain dynamic and competitive framework. Each investment decision involves to determine the timing and the capacity level. In this way we extend the main bulk of the real options theory where the capacity level is given. We consider a monopoly setting as well as a duopoly setting.

Our main results are the following. In the duopoly setting we provide a fully dynamic analysis of entry deterrence/accommodation strategies. We find that the first investor overinvests in capacity in order to delay entry of the second investor. In very uncertain economic environments the first investor always ends up being the largest firm in the market. If uncertainty is moderately present, a reduced value of waiting implies that the preemption mechanism forces the first investor to invest so soon that a large capacity cannot be afforded. Then it will eventually end up with a capacity level being lower than the second investor.

Keywords: Investment under Uncertainty, Entry Deterrence/Accommodation, Duopoly, Capacity Choice

JEL classification: E22,C73,L13

*The authors thank Dolf Talman, Gianluca Femminis, Hettie Boonman, participants of the 13th Annual International Conference on Real Options in Braga (June 2009), the IAS - Investing in an Uncertain Future workshop in Glasgow (September 2009), the NAKRE Research Day in Utrecht (April 2010), the Twelfth Workshop on Optimal Control, Dynamic Games and Nonlinear Dynamics (May 2012), the UT-Austin Portugal workshop in Mathematical Finance and Stochastic Control in Lisbon (July 2012), the Economics of Irreversible Choices workshop in Brescia (September 2012) and seminar participants at the Catholic University of Milan (November 2009), at the Instituto Superior Técnico in Lisbon (January 2010, 2012), and at Aarhus University (November 2011) for their constructive comments.

1 Introduction

When entering a new market it is not only the timing that is important, but also the scale of the investment. By investing at a large scale the firm takes a risk in case of uncertain demand. In particular, revenue may be too low to defray the investment cost if ex-post demand turns out to be disappointingly low. On the other hand, large scale investment gives a high revenue in case of a high demand realization and makes it less attractive for other firms to enter the same market and thus reduce demand for the incumbent firm.

The paper considers a firm's capital investment project where undertaking the investment implies that the firm obtains a production plant. In particular, the firm has to decide when to invest, and, in case it does invest, how much output the production plant is able to produce where the amount increases with the sunk cost investment. This is a real option problem, but, however, the bulk of real option models only determines the optimal timing of an investment project of given size (see Dixit and Pindyck (1994) and Trigeorgis (1996) for an overview). We consider both a monopoly as well as a duopoly setting.

For the single firm case¹, an early contribution by Manne (1961) finds that the firm invests in a larger capacity level when uncertainty increases. Bean *et al.* (1992) generalize Manne (1961) by allowing for nonstationary demand processes as well as general cost structures. Pindyck (1988) studies a single firm model that can make incremental irreversible investments. He finds that irreversibility leads to smaller capacities and that a large portion of the firm value is due to growth possibilities. The starting point of our research is Dixit (1993) (see also Decamps *et al.* (2006)). We obtain that for higher levels of uncertainty the monopolist invests later in a higher quantity (like Manne (1961)), and this was also found in Dangl (1999). The difference in the setup is that Dangl (1999) assumes a slightly different inverse demand curve and furthermore assumes that the firm can adjust its output in downtimes. Bøckman *et al.* (2008) apply the model of Dangl (1999) to study investment in small hydropower projects. Bar-Ilan and Strange (1999) compare lumpy investment with incremental investment. In their lumpy investment setup they find the same as we do, i.e. uncertainty delays investment and increases the size. Hagspiel *et al.* (2012) obtain that production output flexibility makes that it is optimal to invest in a larger capacity level, where uncertainty reinforces this effect. Guthrie (2012) extends this line of research by allowing for an arbitrary number of investments. He finds that greater uncertainty leads firms to undertake infrequent large investments rather than frequent small ones. Guo *et al.* (2005) study the effect of regime shifts on irreversible investments. They find that investment is infinitesimal within a regime and can be lumpy when there is a shift in regime.

In the strategic real option models competition between firms is taken into account. The latter area is surveyed in Grenadier (2000), Huisman *et al.* (2004), Chevalier-Roignant and Trigeorgis (2011), and Azevedo and Paxson (2010). This stream of literature can be divided in two categories: (i) firms that undertake incremental investments and (ii) firms that undertake lumpy investments. Concerning the literature of

¹The single firm case refers either to a monopoly or a firm in a competitive market that does not take into account the actions of its competitors.

incremental investments in a duopoly, one of the early papers is Leahy (1993) that studies investment under uncertainty in a perfect competition setting. Baldursson (1998) and Grenadier (2002) use Leahy (1993)'s myopic solution approach to analyze optimal investment in oligopolistic industries. Aguerrevere (2003) extends these models with time to build and operating flexibility. More recently, Novy-Marx (2007), Back and Paulsen (2009), and Steg (2012) show that the equilibria derived in Grenadier (2002) represent open-loop equilibria. In other words, firms precommit to a strategy. Our model will be solved with feedback strategies, i.e. we derive the closed loop equilibrium.

Lumpy investment decisions are studied in several papers. Early contributions include Smets (1991), Grenadier (1996), Joaquin and Butler (1999), and Weeds (2002). Murto (2004) studies exit in a duopoly setting. Huisman and Kort (2004) analyze a model in which a better investment opportunity becomes available at an uncertain point in time. Boyer *et al.* (2004) investigate firms with multiple investments in case of price competition. Ruiz-Aliseda (2006) studies an entry/exit model in which the market is first increasing and then decreasing. He finds that there will be no entry after the market has started to decline. Bayer (2007) looks at a model in which one firm can increase its capacity to ensure it will exit the market later than its opponent. In a duopoly setting Mason and Weeds (2010) show that the first investment can take place earlier when there is more uncertainty. Boyer *et al.* (2012) present a framework for lumpy capacity building in a duopoly. Grenadier (2002, page 700) advocates lumpy investments by stating that: "... discrete investment is likely to be a more accurate description of reality...". Our paper studies lumpy investments instead of incremental investments. Moreover, whereas in the above mentioned contributions the size of the investment (or divestment) is given for the firms, in our paper the firms can optimally choose the magnitude of the investment.

Our duopoly model provides a dynamic extension of the entry deterrence/accommodation literature being initiated by Spence (1977), Dixit (1979), and Dixit (1980) (see Tirole (1988, Chapter 8) for an excellent explanation of the basic Spence/Dixit model). Asplund (2002) considers risk-averse firms and finds that higher risk aversion only softens competition in the special case that there is marginal cost uncertainty. Maskin (1999) adds uncertainty to the Spence/Dixit model and obtains that the incumbent should hold a higher capacity to deter the entrant. Mason and Phillips (2000) report on the results of experiments on strategic preemption. In case preemption is optimal, players tend to completely preempt. However, they find irrational behavior in case it is partially optimal to preempt. Dewit and Leahy (2006) show that early (aggressive) investment may occur in case firms can commit to the final level of investment. Murto and Pineau (2003) and Murto *et al.* (2004) present models and solution concepts with discrete timesteps and a finite horizon. The former work presents an application to the electricity market and the latter paper illustrates in an example the trade-off between the value of flexibility and economies of scale. Abbring and Campbell (2007) employ a discrete time model to show that if entry barriers exist for new entrants it is possible that the incumbent will serve the total market. Dockner and Mosburger (2007) study a model in discrete time with homogeneous products and they find that firms overinvest and build up large stocks of

capacity to deter the rival from expanding. Besanko *et al.* (2010) study a duopoly model with discrete time and heterogeneous products. Furthermore, there is uncertainty on the rival's exact cost/benefit of capacity addition/withdrawal. They show that preemption races are more likely in case product heterogeneity is low, investment sunkness is low, and depreciation is high. In our model time is continuous, we consider homogeneous products, and we study a duopoly with symmetric firms.

The following two papers are closely related to our work. Wu (2007) also studies a duopoly model in which firms can choose the timing and the size of their investment. In his setup the market is growing until some uncertain point in time and decreases afterwards. It turns out that the first investor will choose a smaller capacity than the second investor. In this way the first investor can make sure that it is better accommodated to the future market decline so that in future it can end up being a monopolist. The only uncertainty in his model is the switching point of market growth to market decline, whereas in our model the market size is subject to uncertainty at each point in time. Yang and Zhou (2007) extend the model of Dangl (1999) to a duopoly in which one of the firms is the incumbent and the other firm is the entrant. Similarly to our paper they show that the incumbent can only deter the entry of the entrant temporarily. Eventually, the second investor enters and a duopoly framework results.

In our model we start with a market where there is no firm active. Furthermore, we identify a domain of market sizes where it is optimal for the first investor to (temporarily) deter entry, and where the accommodation strategy is optimal for the first investor. The difference with Yang and Zhou (2007) is that the latter paper takes the incumbent decision as given, whereas we fully analyze the incumbent's investment decision. In the case of exogenous firm roles, where it is predetermined which firm will invest first, we find that the first investor, after having enjoyed a period of monopoly profits, will end up being the biggest firm in the duopoly market. This result can be turned upside down in case both firms are allowed to invest first. Then the temporary monopoly profits being generated by the first investor create a preemption effect. In case of a moderately uncertain economic environment, the incentive to preempt makes that the first investor invests when the market is still small. This implies that the corresponding capacity level will be relatively low, and this ultimately leads to a duopoly where the second investor can have a larger capacity than the first one. However, when the economic environment is more uncertain, the preemption effect is mitigated by the value of waiting with investment, where the latter arises because of the high uncertainty. Then the first investor's investment is delayed so that it invests when the market is so large that it is profitable to acquire a larger capacity. This always results in a duopoly where the first investor is the larger firm.

Considering welfare we find that the strategic implication of the preemption effect is that the first investor invests too early in a too small capacity. For the second investor, as well as for the monopolist, strategic effects are absent. This results in a welfare optimal timing of investment. However, the corresponding capacity level is still too low from a welfare perspective.

The paper is organized as follows. Section 2 studies the monopoly problem, whereas the duopoly framework is studied in Section 3. Section 4 concludes.

2 Monopoly

We consider a framework with one firm that can undertake an investment to enter a market. The price at time t in this market is given by

$$P(t) = X(t)(1 - \eta Q(t)), \quad (1)$$

where $Q(t)$ is total market output, $\eta > 0$ is a constant, and $X(t)$ is an exogenous shock process. We assume that $X(t)$ follows a geometric Brownian motion:

$$dX(t) = \mu X(t) dt + \sigma X(t) d\omega(t), \quad (2)$$

in which μ is the drift rate, $d\omega(t)$ is the increment of a Wiener process, and $\sigma > 0$ is a constant. The inverse demand function (1) is a special case of, e.g., Dixit and Pindyck (1994, Chapter 9), where they have $P = XD(Q)$ with $D(Q)$ unspecified. Inverse demand being linear in quantity has been adopted also in, e.g., Pindyck (1988), He and Pindyck (1992), Aguerrevere (2003), and Wu (2007). The firm is risk neutral and discounts against rate $r > \mu$.

A firm can become active on this market by investing in capacity. A unit of capacity costs δ . This implies that a firm investing in a plant with capacity Q , incurs investment costs being equal to δQ . We impose that the firm always produces up to capacity. This is because, according to, e.g., Goyal and Netessine (2007), firms may find it difficult to produce below capacity due to fixed costs associated with, for example, labor, commitments to suppliers, and production ramp-up. Even when firms can keep some capacity idle, a temporary suspension of production is often costly. This is the case, for example, because of the maintenance costs needed to avoid deterioration of the equipment. Therefore, in practice firms often reduce prices to keep production lines running.

In Section 2.1 we derive the firm's optimal investment decision, while we study the optimal welfare decision in Section 2.2.

2.1 The Firm's Optimal Investment Decision

Here we study the market entry of a single firm. The corresponding investment problem is solved as an optimal stopping problem in dynamic programming. Let V denote the value of the firm. Then the investment problem that the firm is facing can be formalized as follows:

$$V(X) = \max_{T \geq 0, Q \geq 0} E \left[\int_{t=T}^{\infty} Q X(t) (1 - \eta Q) \exp(-rt) dt - \delta Q \exp(-rT) \middle| X(0) = X \right], \quad (3)$$

where T is the time at which the investment is undertaken, and Q is the quantity or capacity level that the firm acquires at time T . The value of the firm, and thereby the expectation in equation (3), is conditional on the information that is available at time 0. The level of the geometric Brownian motion at that time is set equal to X .

Let X^* be the value of the geometric Brownian motion at which the firm is indifferent between investing and not investing. The corresponding quantity is denoted by $Q^*(X^*)$. For $X > X^*$ we are in the stopping region where it is optimal to invest immediately. When $X < X^*$ demand is (still) too low to undertake the investment. Then we are in the continuation region where the firm thus waits with investing. We study the scenario where $X(0) < X^*$, implying that it is not optimal to invest at the initial point of time.

The optimal investment policy can be found in two steps. First, for a given level of the geometric Brownian motion, denoted by X , the corresponding optimal value of Q is found by solving

$$\max_{Q \geq 0} E \left[\int_{t=0}^{\infty} QX(t) (1 - \eta Q) \exp(-rt) dt - \delta Q \middle| X(0) = X \right], \quad (4)$$

which gives

$$Q^*(X) = \frac{1}{2\eta} \left(1 - \frac{\delta(r - \mu)}{X} \right). \quad (5)$$

From equation (5) we conclude that the optimal capacity level is increasing in X , indicating the level of demand at the moment of investment. At a higher level of X it is profitable for the firm to invest in a larger capacity so that the total profit flow increases. Second, the optimal investment threshold X^* is derived. The next proposition summarizes the results of these two steps. The proofs of all propositions can be found in Appendix A.

Proposition 1 *The value of the monopolist is equal to*

$$V(X) = \begin{cases} AX^\beta & \text{if } X < X^*, \\ \frac{(X - \delta(r - \mu))^2}{4X\eta(r - \mu)} & \text{if } X \geq X^*, \end{cases} \quad (6)$$

in which

$$\beta = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}, \quad (7)$$

$$A = \frac{\delta \left(\frac{\beta+1}{\beta-1} \delta(r - \mu) \right)^{-\beta}}{(\beta^2 - 1)\eta}. \quad (8)$$

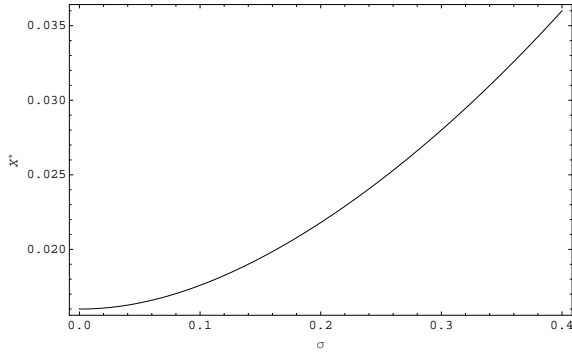
The optimal investment trigger X^* and the corresponding optimal capacity level $Q^*(X^*)$ are given by:

$$X^* = \frac{\beta+1}{\beta-1} \delta(r - \mu), \quad (9)$$

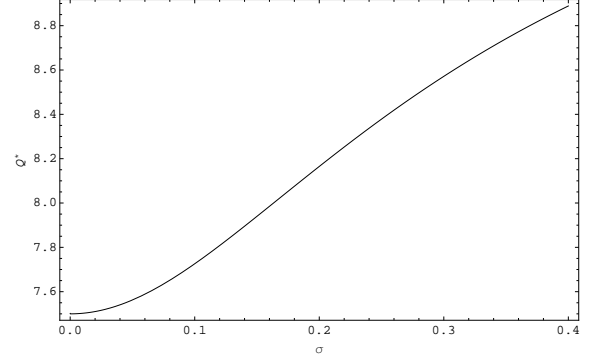
$$Q^* \equiv Q^*(X^*) = \frac{1}{(\beta+1)\eta}. \quad (10)$$

Note that equation (10) is equivalent to equation (8) in Dixit (1993)². Next we carry out some comparative statics analysis. First of all, we have (cf. Dixit and Pindyck (1994)): $\frac{\partial \beta}{\partial \sigma} < 0$, $\frac{\partial \beta}{\partial \mu} < 0$, and $\frac{\partial \beta}{\partial r} > 0$.

²To see this, note that the total investment cost in Dixit (1993) is denoted by K and the profit flow in that paper is equal to PX , where P follows a geometric Brownian motion and X is equal to $F(K)$, being the production function. In our model the total investment cost is equal to δQ and the profit flow is equal to $XQ(1 - \eta Q)$, where X follows a geometric Brownian motion. Equation (13) can be found by taking $F(K) = \left(1 - \eta \frac{K}{\delta}\right) \frac{K}{\delta}$ and maximizing $\frac{(F(K))^\beta}{K^\beta}$ with respect to K as suggested in Dixit (1993).



(a) Investment trigger



(b) Quantity

Figure 1: Optimal investment trigger X^* and optimal quantity Q^* as function of σ . Parameter values are $r = 0.1$, $\mu = 0.06$, $\delta = 0.1$, and $\eta = 0.05$.

Furthermore, differentiating (9) and (10) with respect to β gives

$$\frac{\partial X^*}{\partial \beta} = -\frac{2\delta(r - \mu)}{(\beta - 1)^2} < 0, \quad (11)$$

$$\frac{\partial Q^*}{\partial \beta} = -\frac{1}{((\beta + 1)\eta)^2} < 0. \quad (12)$$

We conclude that, like the standard real options result, increased uncertainty, i.e. a larger value of σ , raises X^* and thus delays investment. However, here we also find that increased uncertainty raises Q^* as well³. This confirms Dixit (1993) who concludes that greater volatility systematically leads to the adoption of larger projects. Figure 1 illustrates these results for a specific example.

2.2 Welfare

To study the welfare implications of the investment timing and size, we first derive the equation for the consumer surplus. Given that the firm is producing with capacity Q , the instantaneous consumer surplus is equal to $\int_{P(Q)}^X D(P) dP$. Since $P(Q) = X(1 - \eta Q)$, we have that $D(P) = \frac{1}{\eta} \left(1 - \frac{P}{X}\right)$, which leads to the following expression for the instantaneous consumer surplus:

$$\int_{X(1-\eta Q)}^X \frac{1}{\eta} \left(1 - \frac{P}{X}\right) dP = \frac{1}{2} X Q^2 \eta. \quad (13)$$

The total expected consumer surplus (CS), given the level X and the capacity Q of the firm, is equal to

$$CS(X, Q) = E \left[\int_{t=0}^{\infty} \frac{1}{2} X(t) Q^2 \eta \exp(-rt) dt \middle| X(0) = X \right] = \frac{X Q^2 \eta}{2(r - \mu)}. \quad (14)$$

³Note that from equation (5) it follows that this is an indirect effect. The increase is caused by the increasing value of X^* .

The expected producer surplus (PS) is equal to the value of the firm, i.e.

$$PS(X, Q) = \frac{XQ(1 - \eta Q)}{r - \mu} - \delta Q. \quad (15)$$

The total expected surplus is the sum of consumer and producer surplus, so that

$$TS(X, Q) = CS(X, Q) + PS(X, Q) = \frac{XQ(2 - \eta Q)}{2(r - \mu)} - \delta Q. \quad (16)$$

Inserting the monopoly decision of Proposition 1, we get that at the moment of investment, the total expected surplus is equal to

$$TS(X^*, Q^*) = \frac{3\delta}{2(\beta + 1)(\beta - 1)\eta}. \quad (17)$$

On the other hand, the social planner, who maximizes total expected surplus, has the following investment threshold and capacity level

$$X_W^* = \frac{\beta + 1}{\beta - 1} \delta(r - \mu) = X^*, \quad (18)$$

$$Q_W^* = \frac{2}{(\beta + 1)\eta} = 2Q^*. \quad (19)$$

In other words, the social planner has the same investment timing as the monopolist. However, the monopolist chooses a capacity level that is half the level of the welfare maximizing strategy. The consumers want prices to be low, so if also consumer surplus is taken into account it makes sense that quantity (or capacity) will be higher implying that the resulting market price is lower. Surprising however, is the fact that investment timing is similar for the monopoly and the social planner case.

The total welfare for the welfare maximizing policy at the moment of investment is equal to

$$TS_W = TS(X_W^*, Q_W^*) = \frac{2\delta}{(\beta + 1)(\beta - 1)\eta}. \quad (20)$$

We conclude that welfare loss in a monopoly situation at the moment of investment equals

$$TS(X_W^*, Q_W^*) - TS(X^*, Q^*) = \frac{\delta}{2(\beta + 1)(\beta - 1)\eta}, \quad (21)$$

which implies that at the start of the planning period the welfare loss is represented by

$$\left(\frac{X(0)}{X^*}\right)^\beta \frac{\delta}{2(\beta + 1)(\beta - 1)\eta} = \frac{(\beta - 1)^{\beta-1} (X(0))^\beta}{2(\beta + 1)^{\beta+1} \delta^{\beta-1} (r - \mu)^\beta \eta}. \quad (22)$$

Here $\left(\frac{X(0)}{X^*}\right)^\beta$ stands for the stochastic discount factor, i.e. it holds that

$$\left(\frac{X(0)}{X^*}\right)^\beta = E[\exp(-rT)],$$

where T is the expected first passage time of reaching X^* (see, e.g., Dixit and Pindyck (1994)), and is thus also the expected investment timing.

The welfare loss decreases with β , which implies that it goes up with uncertainty. Higher uncertainty implies that the monopolist as well as the social planner invests in a large capacity. Since the capacity level

	MONOPOLY			SOCIAL PLANNER			COMPARISON	
σ	X^*	Q^*	TS	X_W^*	Q_W^*	TS_W	$\frac{TS}{TS_W}$	$\frac{Q^*}{Q_W^*}$
0.00	0.01600	7.500	0.01661	0.01600	15.000	0.02215	0.750	0.500
0.05	0.01641	7.563	0.01766	0.01641	15.125	0.02355	0.750	0.500
0.10	0.01759	7.726	0.02069	0.01759	15.452	0.02759	0.750	0.500
0.15	0.01942	7.940	0.02536	0.01942	15.880	0.03382	0.750	0.500
0.20	0.02180	8.165	0.03128	0.02180	16.330	0.04171	0.750	0.500
0.25	0.02467	8.379	0.03806	0.02467	16.757	0.05075	0.750	0.500
0.30	0.02800	8.571	0.04537	0.02800	17.143	0.06049	0.750	0.500
0.35	0.03178	8.741	0.05291	0.03178	17.483	0.07055	0.750	0.500
0.40	0.03600	8.889	0.06048	0.03600	17.778	0.08064	0.750	0.500

Table 1: Welfare results for monopoly. Parameter values are $r = 0.1$, $\mu = 0.06$, $\delta = 0.1$, $\eta = 0.05$, $X(0) = 0.001$.

of the social planner is twice the level of the monopolist, the difference in capacity level also goes up when there is more uncertainty, which leads to a larger welfare loss.

It makes sense that welfare loss will be smaller if parameters change such that both capacity levels will be lower, which happens when the investment becomes more costly or less profitable. This explains why welfare loss is decreasing in δ , r , and η . When $X(0)$ is low, the initial demand level is low too, and it will take a long time before undertaking the investment will be optimal. Then the difference in the total expected surplus resulting from the investment decisions of the monopolist and the social planner is heavily discounted, so that the absolute value of the welfare loss will be small.

Table 1 presents the results of a particular example. We see that there is a welfare loss of 25% which is in line with equations (17) and (21). Furthermore, we see that, indeed, the investment triggers for the monopolist and social planner are equal and that the capacity of the social planner is twice the capacity that the monopolist chooses.

3 Duopoly

This section adds competition to the investment problem in which we determine the optimal timing and capacity of the firm. To do so we extend the model of the previous section by adding an additional firm. We denote by Q_L (Q_F) the capacity of the first (second) investor, so that for the market quantity Q after the two firms have invested it holds that $Q = Q_L + Q_F$. As usual in timing games (see, e.g., Fudenberg and Tirole (1985)), the first investor is called the leader and the second investor the follower. The derivations of the propositions are given in Appendix B.

Section 3.1 studies the game where the firm roles are exogenously assigned. This means that one firm has been given the leader role beforehand. The other firm is the follower, which is not allowed to invest before the leader. As soon as the leader has invested, the follower can choose between investing at the same time or waiting with investment.

The next step is to have endogenous firm roles, i.e. each firm is allowed to invest first. The advantage of being the leader is that, given that the other firm invests later, the leader is a monopolist during the time period between the two investments. This creates an incentive to preempt the other firm with investing. The analysis of this framework is contained in Section 3.2.

3.1 Exogenous firm roles

We assume that the initial demand level is low, i.e. it holds that $X(0)$ falls below any investment trigger derived in this section. The game is solved backwards in time, implying that we start with determining the follower decision. Given that the leader has already invested, the follower cannot influence the investment decision of its competitor anymore, so the follower decision involves no strategic aspects. The follower has to determine the investment timing, which is similar to fixing a threshold level denoted by X_F^* , and the optimal investment capacity Q_F^* . We do this for every given level of the leader's investment capacity, so that we obtain functions $Q_F^*(Q_L)$ and $X_F^*(Q_L)$.

Proposition 2 *Given the current level of the stochastic demand process denoted by X , and the capacity level Q_L of the leader, the optimal capacity level for the follower $Q_F^*(X, Q_L)$ is equal to*

$$Q_F^*(X, Q_L) = \frac{1}{2\eta} \left(1 - \eta Q_L - \frac{\delta(r - \mu)}{X} \right). \quad (23)$$

The value function of the follower $V_F^(X, Q_L)$ is given by*

$$V_F^*(X, Q_L) = \begin{cases} A_F(Q_L) X^\beta & \text{if } X < X_F^*(Q_L), \\ \frac{(X(1 - \eta Q_L) - \delta(r - \mu))^2}{4X\eta(r - \mu)} & \text{if } X \geq X_F^*(Q_L), \end{cases} \quad (24)$$

where

$$X_F^*(Q_L) = \frac{\beta + 1}{\beta - 1} \frac{\delta(r - \mu)}{1 - \eta Q_L}, \quad (25)$$

$$A_F(Q_L) = \left(\frac{(\beta - 1)(1 - \eta Q_L)}{(\beta + 1)\delta(r - \mu)} \right)^\beta \frac{(1 - \eta Q_L)\delta}{(\beta + 1)(\beta - 1)\eta}, \quad (26)$$

so that

$$Q_F^*(Q_L) \equiv Q_F^*(X_F^*(Q_L), Q_L) = \frac{1}{(\beta + 1)\eta} (1 - \eta Q_L). \quad (27)$$

The results of this proposition are similar to those of the monopoly investment problem reported in Proposition 1, except that here the factor $1 - \eta Q_L$ appears in $X_F^*(Q_L)$ and in $Q_F^*(Q_L)$. This is because the leader investment has already taken place, which implies that, according to expression (1), the maximal

output price, i.e. the price that results when $Q_F = 0$, reduces with a factor ηQ_L . This confirms that the follower decision is not influenced by any strategic aspects.

The next step is to determine the investment decision of the leader, where the latter takes the strategy of the follower into account. The follower has two possibilities: investing at the same time as the leader or investing later. Given the current level of X , the leader knows that the follower will invest later if it chooses its capacity Q_L such that $X_F^*(Q_L)$ is larger than X . We refer to this strategy as an entry *deterrence* strategy. Note that from (1) it follows that the output price can become, in principle, infinitely large, so that at one point in time it will always be optimal for the follower to enter, which will happen at the moment that X hits $X_F^*(Q_L)$. Therefore, for any entry deterrence policy it holds that during an initial time period after the investment the leader will be the monopolist. However, this time period will always end at a finite point in time.

From equation (25) it follows that a deterrence strategy occurs whenever the leader chooses a capacity level Q_L larger than $\widehat{Q}_L(X)$, such that

$$\widehat{Q}_L(X) = \frac{1}{\eta} \left(1 - \frac{(\beta + 1) \delta (r - \mu)}{(\beta - 1) X} \right). \quad (28)$$

Hence, in the complementary case, i.e. $Q_L \leq \widehat{Q}_L$, the follower invests at the same time as the leader. In Proposition 3 below we find that the optimal leader capacity $Q_L^{det}(X)$ under entry deterrence is implicitly determined by

$$\frac{X (1 - 2\eta Q_L^{det})}{r - \mu} - \delta - \left(\frac{X (\beta - 1) (1 - \eta Q_L^{det})}{(\beta + 1) \delta (r - \mu)} \right)^\beta \frac{(1 - (\beta + 1) \eta Q_L^{det}) \delta}{(\beta - 1) (1 - \eta Q_L^{det})} = 0. \quad (29)$$

After substitution of $\widehat{Q}_L(X)$ for Q_L^{det} in (29), we can conclude that entry deterrence will not occur if

$$Q_L^{det}(X) < \widehat{Q}_L(X) \Leftrightarrow X > \frac{2(\beta + 1)}{\beta - 1} \delta (r - \mu) = X_2^{det}. \quad (30)$$

Then demand is so high that it is always optimal for the follower to enter immediately once the leader has invested. Note that here, due to the fact that firm roles are exogenous, the rules of the game are such that the designated leader always invests first. This assumption will be relaxed in the next section where both firms are allowed to be the first investor.

An entry deterrence policy for the leader generates the following leader value:

$$V_L^{det}(X, Q_L) = \frac{X (1 - \eta Q_L) Q_L}{r - \mu} - \delta Q_L - \left(\frac{X}{X_F^*(Q_L)} \right)^\beta \frac{X_F^*(Q_L) \eta Q_F^*(Q_L) Q_L}{r - \mu}. \quad (31)$$

The first term stands for the expected total discounted revenue the leader obtains when it is in the market as a monopolist forever. The second term is the initial investment outlay necessary to acquire a production capacity of Q_L . Since the leader is not a monopolist forever, because at some point in time the follower enters and the monopoly turns into a duopoly, the first term needs a negative correction, which is achieved by the third term. Here again $\left(\frac{X}{X_F^*(Q_L)} \right)^\beta$ stands for the stochastic discount factor, i.e. it holds that

$$\left(\frac{X}{X_F^*(Q_L)} \right)^\beta = E[\exp(-rT)], \quad (32)$$

where T is the expected first passage time of reaching $X_F^*(Q_L)$ when $X(0) = X$. As soon as $X(t)$ reaches the threshold $X_F^*(Q_L)$, the follower enters with capacity $Q_F^*(Q_L)$. This reduces the output price with $X_F^*(Q_L) \eta Q_F^*(Q_L)$, and thus instantaneous leader revenue decreases with $X_F^*(Q_L) \eta Q_F^*(Q_L) Q_L$.

To determine the optimal capacity level for a deterrence strategy, the leader maximizes its value (31) with respect to Q_L . This results in the implicit equation (35) given in Proposition 3 below. From this equation it can be derived that the optimal entry deterrence capacity level Q_L^{det} is increasing in X . It follows that by putting Q_L^{det} equal to zero a value for X is found, denoted by X_1^{det} , below which an entry deterrence strategy will not occur. Then the demand level is simply too low for an investment to be profitable. X_1^{det} is implicitly determined by equation (33) in Proposition 3 below, where (33) is derived from putting Q_L^{det} equal to zero in equation (35). The following proposition presents the entry deterrence strategy.

Proposition 3 *The leader will consider the entry deterrence strategy whenever the current level of X is in the interval (X_1^{det}, X_2^{det}) , where X_1^{det} is implicitly defined by*

$$\frac{X_1^{det}}{r - \mu} - \delta - \left(\frac{X_1^{det}(\beta - 1)}{(\beta + 1)\delta(r - \mu)} \right)^\beta \frac{\delta}{\beta - 1} = 0, \quad (33)$$

and

$$X_2^{det} = \frac{2(\beta + 1)}{\beta - 1} \delta(r - \mu). \quad (34)$$

Given that the leader invests at X , the optimal capacity level $Q_L^{det}(X)$ for its entry deterrence strategy is implicitly determined by

$$\frac{X(1 - 2\eta Q_L^{det})}{r - \mu} - \delta - \left(\frac{X(\beta - 1)(1 - \eta Q_L^{det})}{(\beta + 1)\delta(r - \mu)} \right)^\beta \frac{(1 - (\beta + 1)\eta Q_L^{det})\delta}{(\beta - 1)(1 - \eta Q_L^{det})} = 0. \quad (35)$$

The value function for the leader's entry deterrence strategy, when the leader invests at X , $V_L^{det}(X)$, equals

$$V_L^{det}(X) = \frac{X Q_L^{det}(X)(1 - \eta Q_L^{det}(X))}{r - \mu} - \delta Q_L^{det}(X) - \left(\frac{X(\beta - 1)(1 - \eta Q_L^{det}(X))}{(\beta + 1)\delta(r - \mu)} \right)^\beta \frac{\delta Q_L^{det}(X)}{\beta - 1}. \quad (36)$$

Given that X is sufficiently low, i.e. $X \leq X_L^{det}$, for the entry deterrence strategy the optimal investment threshold X_L^{det} and the corresponding quantity Q_L^{det} is given by

$$X_L^{det} = \frac{\beta + 1}{\beta - 1} \delta(r - \mu), \quad (37)$$

$$Q_L^{det}(X_L^{det}) = \frac{1}{(\beta + 1)\eta}. \quad (38)$$

From (37) and (38) we obtain that the leader's investment decision coincides with the one of the monopolist (see Proposition 1). To explain this, first consider the timing decision. Analogous to the standard real options game where capacity is given, the reason is that the timing decision of the leader, i.e. the determination of the threshold X_L^{det} , has no effect on the optimal reply of the follower (see Huisman (2001, p.170)). As far as the capacity level is concerned, note that $Q_L^{det}(X_L^{det})$ is in fact the Stackelberg leader capacity level. In this

framework the leader is committed to produce $Q_L^{det}(X_L^{det})$ at any time after the investment. The follower knows this and therefore adjusts its capacity level accordingly (see (27)). From the industrial organization literature (see, e.g., Tirole (1988, p.315)), we know that for a linear demand schedule the Stackelberg leader quantity is equal to the monopoly quantity, which explains why we obtain that $Q^* = Q_L^{det}(X_L^{det})$.

Alternatively, the leader can apply an entry *accommodation* strategy. Then it chooses its capacity Q_L lower than or equal to $\widehat{Q}_L(X)$, which will trigger the follower to make its investment immediately afterwards. Since the leader is the first firm that undertakes the investment, and is committed to produce up to capacity Q_L after its investment, the leader becomes the Stackelberg leader in the duopoly that is formed after the two investments are undertaken. As with the entry deterrence strategy, there exists an X interval in which the leader will consider this strategy. For low X values the optimal leader quantity in the entry accommodation strategy is too high, i.e. $Q_L^{acc}(X) > \widehat{Q}_L(X)$, to trigger direct follower investment. In other words, there exists an X level, denoted by X_1^{acc} , such that the leader only needs to consider the accommodation strategy for X values larger than X_1^{acc} . The following proposition describes the entry accommodation strategy of the leader.

Proposition 4 *The leader will consider the entry accommodation strategy whenever the current level of X is larger than or equal to X_1^{acc} , where*

$$X_1^{acc} = \frac{\beta + 3}{\beta - 1} \delta(r - \mu). \quad (39)$$

The optimal capacity level Q_L^{acc} for the leader's entry accommodation strategy is given by

$$Q_L^{acc}(X) = \frac{1}{2\eta} \left(1 - \frac{\delta(r - \mu)}{X} \right). \quad (40)$$

The value of the entry accommodation strategy, when the leader invests at X , is equal to

$$V_L^{acc}(X) = \frac{(X - \delta(r - \mu))^2}{8X\eta(r - \mu)}. \quad (41)$$

For X sufficiently small the optimal investment threshold and corresponding capacity level for the entry accommodation strategy are given by

$$X_L^{acc} = \frac{\beta + 1}{\beta - 1} \delta(r - \mu), \quad (42)$$

$$Q_L^{acc}(X_L^{acc}) = \frac{1}{(\beta + 1)\eta}. \quad (43)$$

Since $X_L^{acc} < X_1^{acc}$, the optimal investment threshold X_L^{acc} has in fact no meaning since the demand parameter has to admit at least the value X_1^{acc} before the follower invests at the same time as the leader. In Figure 2 the functions Q_L^{det} , \widehat{Q}_L , and Q_L^{acc} are plotted as function of X . The boundary values X_1^{det} , X_2^{det} , and X_1^{acc} are also depicted. One can see that X_1^{det} is the smallest X value for which the leader chooses a positive capacity level. The follower will invest later than the leader, if the leader chooses a capacity level that is larger than \widehat{Q}_L , i.e. above the dashed line. That is why X_2^{det} is determined by the intersection point

of \hat{Q}_L and Q_L^{det} . For X larger than or equal to X_2^{det} the optimal capacity level of the leader corresponding to the entry deterrence strategy, Q_L^{det} , is smaller than the minimum capacity level needed to generate entry deterrence, \hat{Q}_L , which implies that for sure entry accommodation will occur. Note that in this region it also holds that X is larger than X_1^{acc} . On the other hand, for X smaller than X_1^{acc} the optimal capacity level of the leader associated with the entry accommodation strategy, Q_L^{acc} , is larger than the capacity level that ensures direct entry of the follower, \hat{Q}_L , so that only entry deterrence is a possible strategy.

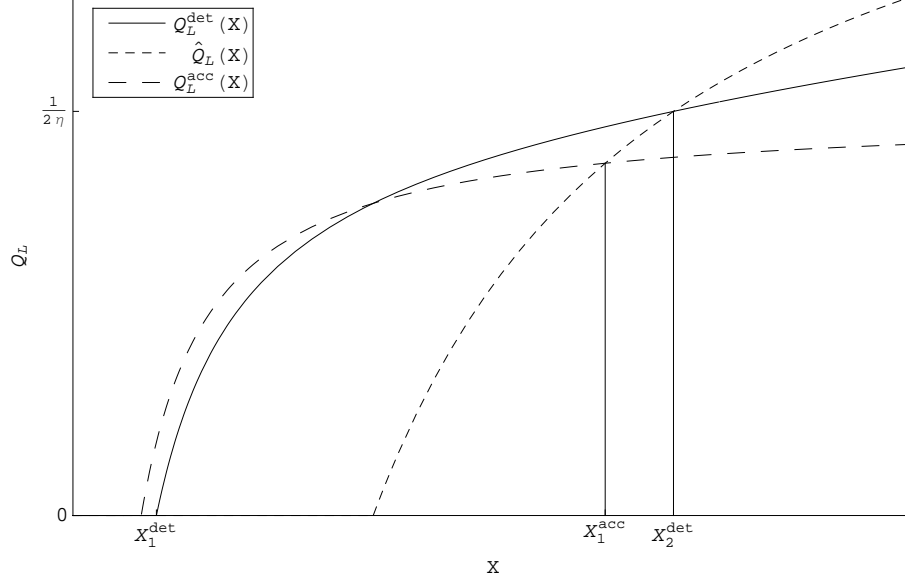


Figure 2: Q_L^{det} , \hat{Q}_L , and Q_L^{acc} as function of X .

We conclude that for X less than X_1^{acc} the leader for sure applies an entry deterrence policy and for X larger than X_2^{det} the leader can only choose for entry accommodation. For $X \in (X_1^{acc}, X_2^{det})$ either the entry deterrence or the entry accommodation strategy maximizes the leader value. We also know that, in case X is less than X_L^{det} , the leader will wait with investment until X hits X_L^{det} for the first time. Combining these observations we arrive at the following proposition, which describes the optimal leader strategy.

Proposition 5 *The optimal capacity level of the leader satisfies*

$$Q_L^*(X) = \begin{cases} Q_L^{det}(X_L^{det}) & \text{if } X \in [0, X_L^{det}), \\ Q_L^{det}(X) & \text{if } X \in [X_L^{det}, \hat{X}), \\ Q_L^{acc}(X) & \text{if } X \in [\hat{X}, \infty), \end{cases} \quad (44)$$

where \hat{X} is such that

$$\hat{X} = \min \{ X \in (X_1^{acc}, X_2^{det}) \mid V_L^{acc}(X) = V_L^{det}(X) \}. \quad (45)$$

The value of the leader is given by

$$V_L^*(X) = \begin{cases} \left(\frac{X}{X_L^{det}}\right)^\beta V_L^{det}(X_L^{det}) & \text{if } X \in [0, X_L^{det}), \\ V_L^{det}(X) & \text{if } X \in [X_L^{det}, \hat{X}), \\ V_L^{acc}(X) & \text{if } X \in [\hat{X}, \infty). \end{cases} \quad (46)$$

The following proposition gives the optimal investment threshold for the exogenous leader.

Proposition 6 *The investment threshold for the exogenous leader is equal to*

$$X_L^* = \begin{cases} X_L^{det} & \text{if } X \in [0, X_L^{det}), \\ X & \text{if } X \in [X_L^{det}, \infty). \end{cases} \quad (47)$$

The optimal capacity level for the leader $Q_L^*(X)$ and the optimal capacity level of the follower $Q_F^*(X)$ are plotted in Figure 3. From (27) and (38) it follows that already at the threshold X_L^{det} the leader invests in a capacity level so high that it exceeds the capacity level that the follower will invest in later. Hence, the leader will be the bigger firm on the market after the follower has invested. Figure 3 shows that if the initial level of X is larger than X_L^{det} , but still lower than \hat{X} , the leader's optimal capacity level $Q_L^{det}(X)$ is increasing in X . The latter characteristic can formally be obtained from (35) that implicitly determines $Q_L^{det}(X)$. In the entry deterrence region (X_L^{det}, \hat{X}) the leader overinvests in capacity in order to make it less attractive for the follower to invest. The implication is that the follower will postpone its investment, which lengthens the period that the leader enjoys monopoly profits. In Figure 3 this overinvestment is visualized by the difference in capacity levels between entry deterrence and entry accommodation at \hat{X} (note that the latter strategy is applied for $X > \hat{X}$). The capacity level corresponding to the entry accommodation strategy is the Stackelberg leader capacity level. This implicitly proves that the entry deterrence capacity level at \hat{X} exceeds the Stackelberg leader capacity level, which confirms the overinvestment property. Intuitively overinvestment results, because, in addition to the fact that higher leader capacity reduces the follower's capacity (see (27)), it also holds that a higher leader capacity will delay entry of the follower (see (25)). The first effect is already taken into account when determining the Stackelberg leader capacity level. However, this does not hold for the second effect, i.e. the effect that a higher leader capacity delays entry of the follower.

Figure 4 shows the value functions for the leader, V_L^* , and the follower, V_F^* . The leader value is the value corresponding to the leader payoff after immediate investment, taking into account that the follower will invest at its optimal threshold level. For small values of X demand is too low for immediate investment to be optimal, which explains why the leader value falls below the follower value there. At the moment the leader switches from an entry deterrence to an entry accommodation strategy, which happens at $X = \hat{X}$, it reduces its capacity level (see Figure 3). Then output price goes up for the follower, which explains the upward jump of the follower value at $X = \hat{X}$.

The following proposition presents the effect of uncertainty on the entry deterrence and entry accommodation domains.

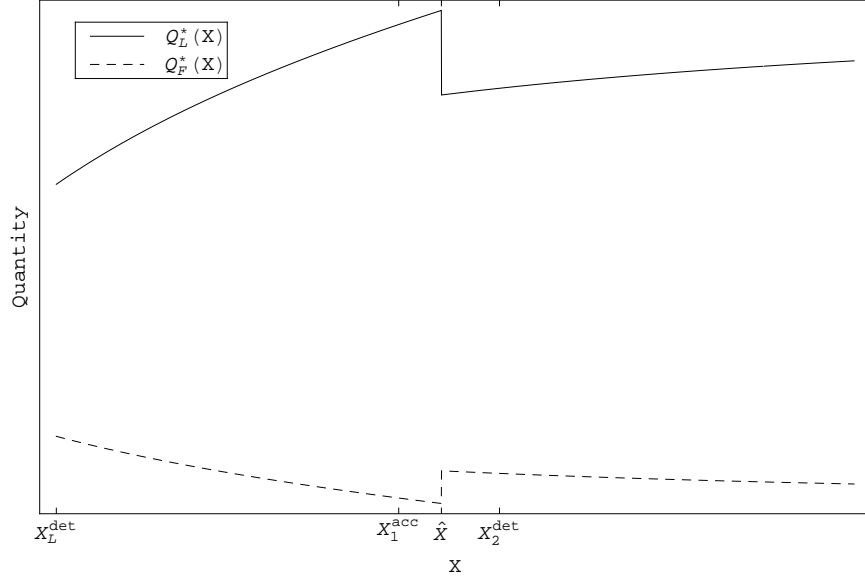


Figure 3: Optimal quantities for the leader, Q_L^* , and the follower, Q_F^* , as function of X .

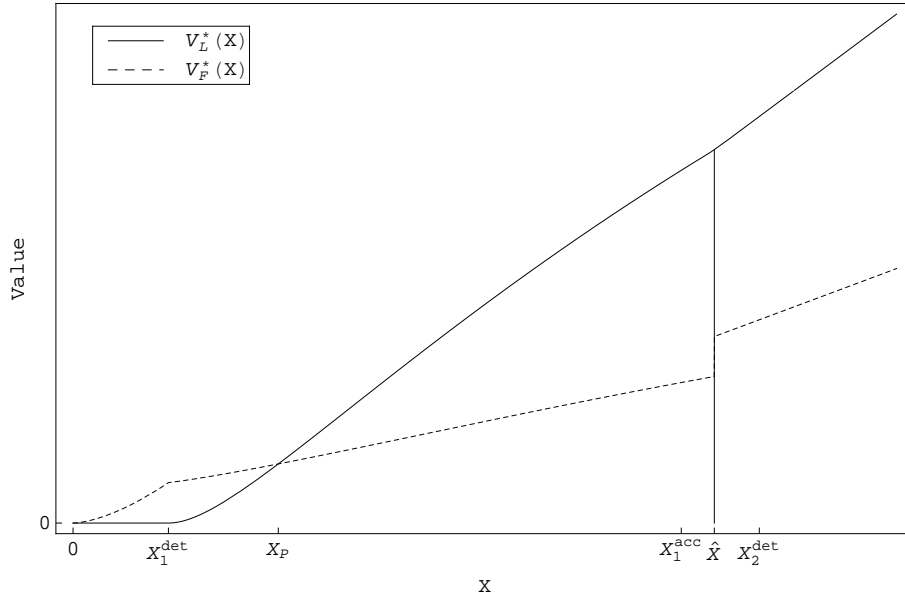


Figure 4: Optimal value functions for the leader (V_L^*) and follower (V_F^*) as function of the X at which the leader invests.

Proposition 7 *The strategy boundaries X_1^{det} , X_2^{det} , and X_1^{acc} are increasing with uncertainty. Furthermore, the region in which the leader can choose between entry deterrence and entry accommodation, $X \in (X_1^{acc}, X_2^{det})$, decreases with uncertainty.*

Since X_1^{acc} is increasing with uncertainty, we conclude that the X interval where only entry deterrence occurs, becomes bigger in uncertain economic environments. The reason is the standard result in real options theory that the value of waiting increases with uncertainty. Due to this the follower has an incentive to invest later when uncertainty goes up. This also makes it more attractive for the leader to apply entry deterrence, because the leader will then enjoy a longer time period in which it is the monopolist in the market.

3.2 Endogenous firm roles

This section employs the knowledge of the previous section to analyze the model with endogenous firm roles. In this scenario the leader and follower roles are not assigned beforehand, so that both firms are allowed to invest first. The firms have an incentive to preempt each other, because after the investment the first investor will be the monopolist in the market. This will last until the other firm invests. Since at the time this other firm invests, the first investor has already invested, there are no strategic aspects related to this investment decision. Consequently, the timing and capacity level of the investment of the second investor are the same as the ones corresponding to the investment of the follower in the exogenous firm roles case of the previous section. Hence, Proposition 2 carries over to the framework analyzed in this section.

Next, we turn to the investment decision of the first investor, who invests at the so-called preemption trigger denoted by X_P , given that the demand level at time zero is such that $X(0) < X_P$. When determining this trigger we depart from the literature on timing games where firms just decide about timing so that the capacity level is exogenously given. A seminal paper in this field is Fudenberg and Tirole (1985), where the investment decisions of two firms are studied within a deterministic framework. This analysis is extended to stochastic timing games in, e.g., Thijssen *et al.* (2012). From this literature we obtain that the preemption trigger X_P is the solution of the following equation:

$$V_L^*(X_P) = V_F^*(X_P, Q_L^*(X_P)). \quad (48)$$

The intuition behind (48) is that whenever $X < X_P$, the payoff of the second investor is larger, since the demand level is too small for an investment to be undertaken. It follows that no firm wants to invest in such a case. On the other hand, when $X > X_P$ the payoff of the first investor is larger. Hence, given that firm 1 tries to invest at this X , it is optimal for firm 2 to preempt by investing at $X - \varepsilon$, which induces firm 1 to invest at $X - 2\varepsilon$. This preemption mechanism continues until $X - n\varepsilon = X_P$, where one of the firms will actually invest. Note that (48) implies that at this point the firms are indifferent between being the first or the second investor, since the resulting payoffs are the same. The first investor (leader) invests at X_P in capacity $Q_L^*(X_P)$ and the second investor (follower) invests at $X_F^*(Q_L^*(X_P))$ in capacity $Q_F^*(Q_L^*(X_P))$.

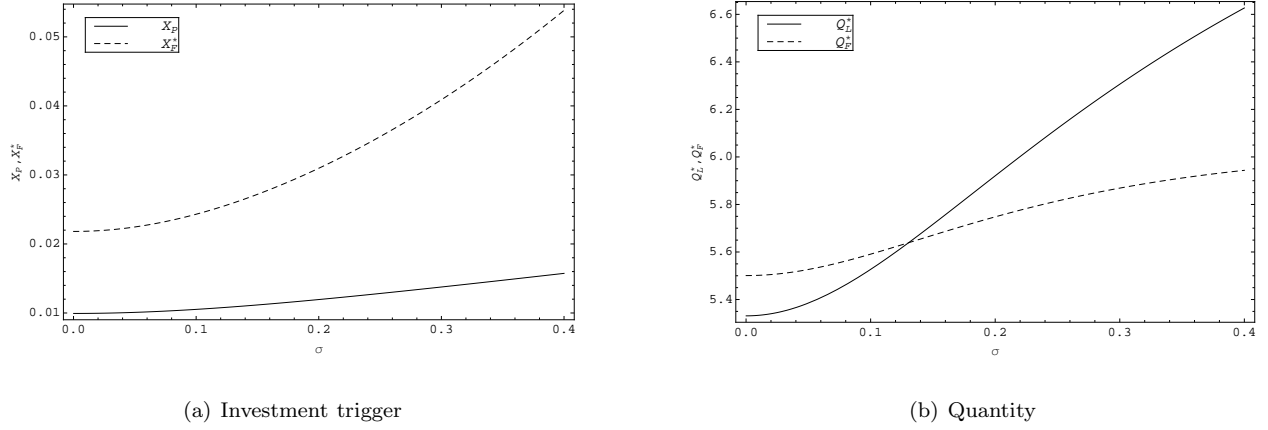


Figure 5: Equilibrium investment triggers for the leader, X_P , and follower, X_F^* , and corresponding capacities Q_L^* and Q_F^* as function of σ . Parameter values are $r = 0.1$, $\mu = 0.06$, $\delta = 0.1$, and $\eta = 0.05$.

One of the main results of the previous section on exogenous firm roles is that the leader capacity is always bigger than the follower capacity. The fact that we do not have an explicit expression for X_P (instead, X_P is implicitly defined by (48)), makes it impossible to obtain analytical results regarding equilibrium capacity levels in the endogenous firm roles case. However, extensive numerical experiments lead to the result that the investment threshold for the leader (X_P) (as well as for the follower (X_F^*)) is increasing with uncertainty. Due to the preemption threat the leader can be forced to invest so early that at the moment of investment the market is too small to invest in a large capacity. The implication can be that the follower's capacity will be bigger in equilibrium. Figure 5 illustrates this result for one particular example. This happens for low values of uncertainty. A high level of uncertainty generates a value of waiting. This delays investment with the implication that at the moment of investment of the leader the market has grown enough to invest in a large capacity. Then in equilibrium the leader is again the bigger firm. Figure 6 shows that this result does not hold in general, i.e. it can also be the case that the leader is already the larger firm for low values of uncertainty.

3.3 Welfare

In order to analyze the duopoly investment outcome from a welfare perspective, we consider a social planner that is allowed to undertake an investment at two moments in time. For both investments the social planner can freely choose the timing and the capacity level. The investment strategy that maximizes welfare is determined backwards in time. First, conditionally on the capacity level of the first investment, we establish the investment trigger and capacity of the second investment. After that we determine timing and size of the first investment.

Concerning the second investment we know that the investment decision of the follower in the duopoly

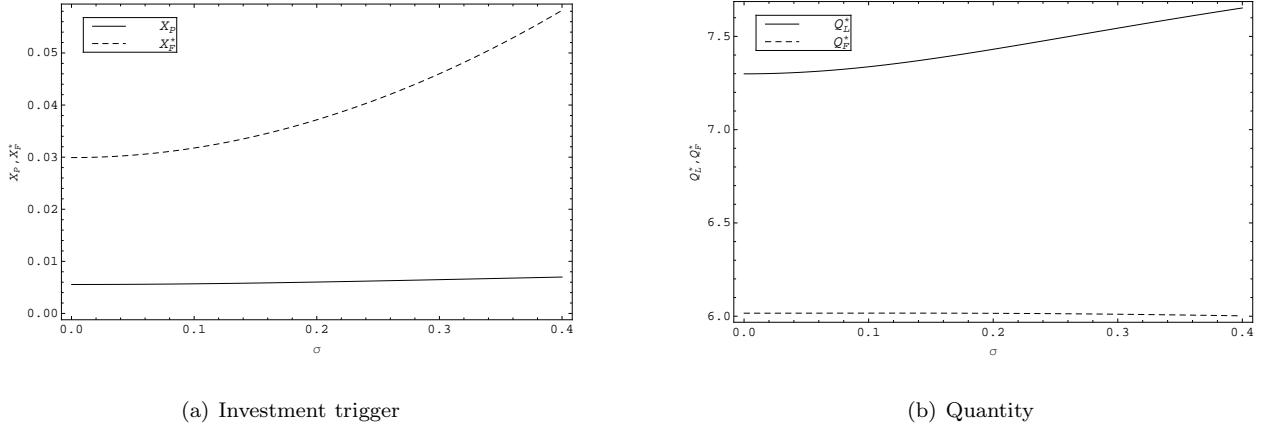


Figure 6: Equilibrium investment triggers for the leader, X_P , and follower, X_F^* , and corresponding capacities Q_L^* and Q_F^* as function of σ . Parameter values are $r = 0.1$, $\mu = 0.09$, $\delta = 0.1$, and $\eta = 0.05$.

model is essentially the same as the investment decision of the monopolist. Consequently, as in the monopoly model of Section 2, it holds that, when we assume for the moment that capacity levels associated with the first welfare investment and the leader investment in the duopoly model are the same, the optimal capacity of the second investment in the welfare maximizing policy, which is denoted by $Q_{F,W}^*$, will be twice as high as the capacity that the follower chooses. The investment timing that corresponds to the investment trigger denoted by $X_{F,W}^*$. Hence, it holds that

$$Q_{F,W}^*(Q_L) = \frac{2(1 - \eta Q_L)}{(\beta + 1)\eta}, \quad (49)$$

$$X_{F,W}^*(Q_L) = \frac{\beta + 1}{\beta - 1} \frac{(r - \mu)\delta}{1 - \eta Q_L}. \quad (50)$$

Concerning the first investment we have that the welfare maximizing capacity, which is denoted by $Q_{L,W}^*$, is twice as high as the capacity that the monopolist chooses in case it can make two investments (see Appendix B for the details of this case). This capacity level is implicitly defined by

$$\frac{1 - \frac{1}{2}\eta Q_{L,W}^*(\beta + 1)}{1 - \frac{1}{2}\eta Q_{L,W}^*} - 2 \left(\frac{\beta(1 - \eta Q_{L,W}^*)}{(\beta + 1)(1 - \frac{1}{2}\eta Q_{L,W}^*)} \right)^\beta = 0. \quad (51)$$

The welfare maximizing trigger of the first investor is given by

$$X_{L,W}^*(Q_L) = \frac{\beta(r - \mu)\delta}{(\beta - 1)(1 - \frac{1}{2}\eta Q_L)}. \quad (52)$$

We conclude that, if capacity levels are the same, the investment triggers of the welfare maximizing strategy are the same as the investment triggers of the monopolist that can invest twice. However, the welfare maximizing capacities are twice as high as the capacities that the monopolist chooses, since private firms do not take consumer surplus into account when deciding about investment. According to (49) and

(50) this implies that the second welfare investment will take place later than the follower investment in the duopoly model, while the corresponding capacity level will be less than twice as high. Furthermore, in the duopoly with endogenous firm roles the firms invest too early in too small capacities from a welfare perspective.

The total expected welfare TS depends on the investment moments, X_L and X_F , the chosen capacities of the leader and the follower, Q_L and Q_F , and the level of the geometric Brownian motion, X , and is given by

$$\begin{aligned} TS(X_L, Q_L, X_F, Q_F, X) &= \left(\frac{X}{X_L}\right)^\beta \left(\frac{X_L Q_L (2 - \eta Q_L)}{2(r - \mu)} - \delta Q_L\right) \\ &+ \left(\frac{X}{X_F}\right)^\beta \left(\frac{X_F (Q_L + Q_F) (2 - \eta (Q_L + Q_F))}{2(r - \mu)} - \delta Q_F - \frac{X_F Q_L (2 - \eta Q_L)}{2(r - \mu)}\right). \end{aligned} \quad (53)$$

Table 2 presents the welfare results for a specific numerical example. The parameter values are equal to those of the example in Section 2. As expected we find that the welfare loss in the duopoly (around 12%) is less than the welfare loss in the monopoly (25 %). Furthermore, we see that the welfare loss is first slightly decreasing with uncertainty and after that increasing with uncertainty. Two contradictory effects play a role here. On the one hand, welfare loss increases with uncertainty, because, as uncertainty goes up the social planner delays investment much more than the firms in the duopoly. Hence, the social value of waiting is higher than the private one. This is caused first by the larger capacity the social planner invests in, and, second, the preemption mechanism in the duopoly mitigates the value of waiting of the first investor. On the other hand the welfare loss decreases with uncertainty, because all investments will take place at a later point in time, so that discounting reduces differences in welfare levels.

From the last column of Table 2 we conclude that, compared to the welfare maximizing capacity, the total capacity in the market in the duopoly increases with uncertainty and is closer to the welfare maximizing capacity (63%-66%) than in the monopoly case (50%).

The capacity that the social planner chooses for the follower ($Q_{F,W}^*$) is first increasing and then decreasing in uncertainty. Again there are two contradictory effects. On the one hand, when uncertainty goes up the value of waiting increases, so that the first investment takes place at a later point of time. Then demand is higher so that it is optimal to invest in a higher capacity too. This implies that the second investment becomes less profitable so that the firm will then invest in less capacity. On the other hand, when uncertainty goes up also the second investment will take place at a later point of time when demand will be higher. This positively influences the capacity level.

4 Conclusions

The paper employs a duopoly framework to extend the static Industrial Organization literature regarding entry deterrence/accommodation strategies to a dynamic uncertain environment. In such a framework firms have to take into account incentives to preempt, because the first investor enjoys some period with monopoly

	DUOPOLY					SOCIAL PLANNER					COMPARISON	
σ	X_P	Q_L^*	X_F^*	Q_F^*	TS	$X_{L,W}^*$	$Q_{L,W}^*$	$X_{F,W}^*$	$Q_{F,W}^*$	TS_W	$\frac{TS}{TS_W}$	$\frac{Q_L^*+Q_F^*}{Q_{L,W}^*+Q_{F,W}^*}$
0.00	0.00993	5.331	0.02182	5.501	0.02065	0.01291	9.010	0.02912	8.242	0.02352	0.87818	0.62785
0.05	0.01009	5.385	0.02246	5.526	0.02193	0.01321	9.086	0.03007	8.254	0.02497	0.87826	0.62927
0.10	0.01052	5.527	0.02431	5.591	0.02561	0.01406	9.284	0.03283	8.279	0.02915	0.87842	0.63302
0.15	0.01116	5.716	0.02719	5.671	0.03125	0.01538	9.544	0.03714	8.302	0.03557	0.87852	0.63807
0.20	0.01195	5.921	0.03096	5.748	0.03834	0.01709	9.817	0.04281	8.315	0.04365	0.87845	0.64355
0.25	0.01283	6.120	0.03555	5.815	0.04641	0.01916	10.076	0.04972	8.315	0.05285	0.87818	0.64895
0.30	0.01376	6.306	0.04089	5.869	0.05504	0.02156	10.310	0.05779	8.306	0.06271	0.87771	0.65401
0.35	0.01474	6.475	0.04699	5.911	0.06389	0.02427	10.516	0.06701	8.290	0.07285	0.87708	0.65863
0.40	0.01574	6.627	0.05384	5.944	0.07271	0.02730	10.695	0.07738	8.271	0.08297	0.87631	0.66279

Table 2: Welfare results for duopoly. Parameter values are $r = 0.1$, $\mu = 0.06$, $\delta = 0.1$, $\eta = 0.05$, $X(0) = 0.001$.

profits, and a value of waiting with investment induced by uncertainty. We show that entry can only be temporary deterred, because at one point in time the market will have grown sufficiently for the second firm to enter. We find that the first investor will overinvest not only to tempt the second investor to invest in a smaller capacity but also to let the second investor invest later. In a moderately uncertain environment the preemption effect dominates, implying that the first investor invests early in a small capacity, so that in the end the second investor can be the larger firm. However, the value of waiting effect dominates in highly uncertain economic environments. Then the first investor invests relatively late in a larger capacity level, implying that it will be the larger firm in the market after the second firm has also invested.

A limitation of our model is that firms can invest only once, i.e. a once installed capacity cannot be extended later on. This may influence our result that the first investor ends up being the smaller firm in the end when uncertainty is moderately present. On the other hand one can argue that an entrant (the second investor) has more incentive to invest in additional market capacity than the incumbent (the first investor), which is due to Arrow's replacement effect: if an incumbent invests it increases capacity and thus market quantity. This reduces the output price and thus profitability of the existing capacity, leading to a reduction of the profitability of the incumbent's investment opportunity.

One other assumption is that firms have to produce up to capacity. Dixit (1980) proves that in a static setting entry deterrence is largely ineffective if firms cannot commit to produce at full capacity. It might be interesting to examine whether Dixit's result is still true in a stochastic dynamic setting. In particular, it would be interesting to establish the role of uncertainty here.

A Proofs of Propositions

Proof of Proposition 1 The profit of this firm at time t is denoted by $\pi(t)$ and is equal to

$$\pi(t) = P(t)Q(t) = X(t)Q(t)(1 - \eta Q(t)). \quad (54)$$

We denote by $V(X, Q)$ the expected value of the firm at the moment of investment given that the current level of $X(t)$ is X and the firm invests in Q units of capital. Then it holds that

$$V(X, Q) = E \left[\int_{t=0}^{\infty} \pi(t) \exp(-rt) dt - \delta Q \right] = \frac{XQ(1 - \eta Q)}{r - \mu} - \delta Q. \quad (55)$$

Maximizing with respect to Q gives the optimal capacity size Q^* for every given level of X :

$$Q^*(X) = \frac{1}{2\eta} \left(1 - \frac{\delta(r - \mu)}{X} \right). \quad (56)$$

Standard real options analysis (e.g. Dixit and Pindyck (1994)) shows that the value of the option to invest, denoted by F , is equal to

$$F(X) = AX^\beta, \quad (57)$$

where β is the positive root of the quadratic polynomial

$$\frac{1}{2}\sigma^2\beta^2 + \left(\mu - \frac{1}{2}\sigma^2\right)\beta - r = 0, \quad (58)$$

and is thus given by

$$\beta = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}. \quad (59)$$

To determine the indifference level X^* we employ the value matching and smooth pasting conditions:

$$F(X^*) = V(X^*, Q), \quad (60)$$

$$\left. \frac{\partial F(X)}{\partial X} \right|_{X=X^*} = \left. \frac{\partial V(X, Q)}{\partial X} \right|_{X=X^*}. \quad (61)$$

Substitution of (55) and (56) into (60) and (61) and solving for X^* gives

$$X^*(Q) = \frac{\beta\delta(r - \mu)}{(\beta - 1)(1 - \eta Q)}. \quad (62)$$

From (56) and (62) we obtain the results.

Proof of Proposition 2 The value function of the follower at the moment of investment is denoted by V_F^* , it depends on X , Q_L , and Q_F , and is equal to

$$V_F^*(X, Q_L, Q_F) = \frac{XQ_F(1 - \eta(Q_L + Q_F))}{r - \mu} - \delta Q_F. \quad (63)$$

Maximizing with respect to Q_F gives the optimal capacity size of the follower, given the level X and the capacity size of the leader Q_L :

$$Q_F^*(X, Q_L) = \frac{1}{2\eta} \left(1 - \eta Q_L - \frac{\delta(r - \mu)}{X} \right). \quad (64)$$

Before the follower has invested, thus when $X < X_F^*(Q_L)$, the firm holds an option to invest. The option value is

$$F_F(X) = A_F X^\beta. \quad (65)$$

Solving the corresponding value matching and smooth pasting conditions gives

$$X_F^*(Q_L, Q_F) = \frac{\beta}{\beta - 1} \frac{\delta(r - \mu)}{(1 - \eta(Q_L + Q_F))}, \quad (66)$$

$$A_F(Q_L) = \left(\frac{(\beta - 1)(1 - \eta Q_L)}{(\beta + 1)\delta(r - \mu)} \right)^\beta \frac{(1 - \eta Q_L)\delta}{(\beta + 1)(\beta - 1)\eta}. \quad (67)$$

We conclude that (after solving the system of equations (64) and (66))

$$X_F^*(Q_L) = \frac{\beta + 1}{\beta - 1} \frac{\delta(r - \mu)}{1 - \eta Q_L}, \quad (68)$$

$$Q_F^*(Q_L) = \frac{1 - \eta Q_L}{(\beta + 1)\eta}. \quad (69)$$

Proof of Proposition 3 The value function of the leader at the moment of investment for the deterrence strategy is given by

$$V_L^{det}(X, Q_L) = \frac{X Q_L (1 - \eta Q_L)}{r - \mu} - \delta Q_L - \left(\frac{X}{X_F^*(Q_L)} \right)^\beta \left(\frac{X_F^*(Q_L) Q_L \eta Q_F^*(Q_L)}{r - \mu} \right). \quad (70)$$

Substitution of (68) and (69) into this equation results in

$$V_L^{det}(X, Q_L) = \frac{X Q_L (1 - \eta Q_L)}{r - \mu} - \delta Q_L - \left(\frac{X (\beta - 1) (1 - \eta Q_L)}{(\beta + 1) \delta (r - \mu)} \right)^\beta \frac{\delta Q_L}{\beta - 1}. \quad (71)$$

Maximizing with respect to Q_L gives the following first order condition:

$$\phi(X, Q_L) \equiv \frac{X (1 - 2\eta Q_L)}{r - \mu} - \delta - \left(\frac{X (\beta - 1) (1 - \eta Q_L)}{(\beta + 1) \delta (r - \mu)} \right)^\beta \frac{(1 - (\beta + 1) \eta Q_L) \delta}{(\beta - 1) (1 - \eta Q_L)} = 0. \quad (72)$$

Solving (72) gives $Q_L^{det}(X)$. Setting $Q_L = 0$ in equation (72) gives equation (33). Define

$$\psi(X) = \frac{X}{r - \mu} - \delta - \left(\frac{X (\beta - 1)}{(\beta + 1) \delta (r - \mu)} \right)^\beta \frac{\delta}{\beta - 1}, \quad (73)$$

then we have that

$$\psi(0) = -\delta < 0, \quad (74)$$

$$\psi(X_F^*(0)) = \frac{\delta}{\beta - 1} > 0, \quad (75)$$

$$\frac{\partial \psi(X)}{\partial X} = \frac{1}{r - \mu} \left(1 - \frac{\beta}{\beta + 1} \left(\frac{X (\beta - 1)}{(\beta + 1) \delta (r - \mu)} \right)^{\beta - 1} \right). \quad (76)$$

For $X \in (0, X_F^*(0))$ it holds that

$$\frac{\partial \psi(X)}{\partial X} > 0, \quad (77)$$

so that we have shown that X_1^{det} exists. Furthermore, the leader cannot use the deterrence strategy anymore if we have that

$$X_F^*(Q_L^{det}(X)) \leq X. \quad (78)$$

Let us define X_2^{det} as

$$X_F^*(Q_L^{det}(X_2^{det})) = X_2^{det}. \quad (79)$$

To determine X_2^{det} we substitute equation (68) for X into (73):

$$\frac{\frac{\beta + 1}{\beta - 1} \frac{\delta(r - \mu)}{1 - \eta Q_L} (1 - 2\eta Q_L)}{r - \mu} - \delta - \left(\frac{\frac{\beta + 1}{\beta - 1} \frac{\delta(r - \mu)}{1 - \eta Q_L} (\beta - 1) (1 - \eta Q_L)}{(\beta + 1) \delta (r - \mu)} \right)^\beta \frac{(1 - (\beta + 1) \eta Q_L) \delta}{(\beta - 1) (1 - \eta Q_L)} = 0. \quad (80)$$

Some rearrangement gives

$$\frac{\beta + 1}{\beta - 1} \frac{1 - 2\eta Q_L}{1 - \eta Q_L} - \delta - \frac{(1 - (\beta + 1) \eta Q_L) \delta}{(\beta - 1) (1 - \eta Q_L)} = 0, \quad (81)$$

so that

$$Q_L = \frac{1}{2\eta}. \quad (82)$$

Substitution of (82) into (68) gives

$$X_2^{det} = \frac{2(\beta+1)}{\beta-1} \delta(r-\mu). \quad (83)$$

Before the leader has invested, thus when $X < X_L^{det}$, the firm holds an option to invest. The option value is

$$F_L^{det}(X) = A_L^{det} X^\beta. \quad (84)$$

The value matching and smoothing pasting conditions to determine X_L^{det} are given by

$$A_L^{det} X^\beta = \frac{X Q_L(X) (1 - \eta Q_L(X))}{r - \mu} - \delta Q_L(X) - \left(\frac{X(\beta-1)(1-\eta Q_L(X))}{(\beta+1)\delta(r-\mu)} \right)^\beta \frac{\delta Q_L(X)}{\beta-1} \quad (85)$$

$$\begin{aligned} \beta A_L^{det} X^{\beta-1} &= \frac{Q_L(X) (1 - \eta Q_L(X)) + X \frac{\partial Q_L}{\partial X} (1 - 2\eta Q_L(X))}{r - \mu} - \delta \frac{\partial Q_L}{\partial X} \\ &\quad - \left(\frac{X(\beta-1)(1-\eta Q_L(X))}{(\beta+1)\delta(r-\mu)} \right)^\beta \delta \frac{\left(Q_L(X) (\beta(1-\eta Q_L(X)) - (\beta+1)X\eta \frac{\partial Q_L}{\partial X}) + X \frac{\partial Q_L}{\partial X} \right)}{X(\beta-1)(1-\eta Q_L(X))}. \end{aligned} \quad (86)$$

Substitution of (86) into (85) gives

$$\begin{aligned} &\frac{X Q_L(X) (1 - \eta Q_L(X))}{r - \mu} - \frac{X Q_L(X) (1 - \eta Q_L(X)) + X^2 \frac{\partial Q_L}{\partial X} (1 - 2\eta Q_L(X))}{\beta(r-\mu)} \\ &- \delta Q_L(X) + \frac{\delta X}{\beta} \frac{\partial Q_L}{\partial X} \\ &+ \left(\frac{X(\beta-1)(1-\eta Q_L(X))}{(\beta+1)\delta(r-\mu)} \right)^\beta \left(\frac{\delta X \frac{\partial Q_L}{\partial X} (1 - (\beta+1)\eta Q_L(X))}{\beta(\beta-1)(1-\eta Q_L(X))} \right) \\ &= 0. \end{aligned} \quad (87)$$

Rewriting equation (80) gives

$$\left(\frac{\frac{\beta+1}{\beta-1} \frac{\delta(r-\mu)}{1-\eta Q_L} (\beta-1)(1-\eta Q_L)}{(\beta+1)\delta(r-\mu)} \right)^\beta = \left(\frac{\frac{\beta+1}{\beta-1} \frac{\delta(r-\mu)}{1-\eta Q_L} (1-2\eta Q_L)}{r-\mu} - \delta \right) \frac{(\beta-1)(1-\eta Q_L)}{(1-(\beta+1)\eta Q_L)\delta}. \quad (88)$$

Substitution of (88) into equation (87) and rewriting gives

$$\frac{X Q_L (1 - \eta Q_L)}{r - \mu} (\beta - 1) - \delta Q_L \beta = 0. \quad (89)$$

So that the leader threshold X_L^{det} is given by

$$X_L^{det} = \frac{\beta+1}{\beta-1} \delta(r-\mu). \quad (90)$$

The corresponding quantity Q_L^{det} can be calculated by substituting equation (90) into equation (72):

$$Q_L^{det} = \frac{1}{(\beta+1)\eta}. \quad (91)$$

Proof of Proposition 4 For the accommodation strategy the value function of the leader is given by

$$V_L^{acc}(X, Q_L) = \frac{X Q_L (1 - \eta(Q_L + Q_F^*(Q_L)))}{r - \mu} - \delta Q_L. \quad (92)$$

Substitution of (64) into (92) and maximizing with respect to Q_L gives

$$Q_L^{acc}(X) = \frac{1}{2\eta} \left(1 - \frac{\delta(r-\mu)}{X} \right). \quad (93)$$

Equation (41) is the result of the substitution of equation (93) into equation (92). The leader will only use its accommodation strategy if the optimal quantity $Q_L^{acc}(X)$ leads to immediate investment of the follower. So it should hold that

$$X_F^*(Q_L^{acc}(X)) \leq X. \quad (94)$$

We define X_1^{acc} as

$$X_1^{acc} = X_F^*(Q_L^{acc}(X_1^{acc})). \quad (95)$$

Substitution of (68) and (93) into (95) and rearranging gives

$$X_1^{acc} = \frac{\beta+3}{\beta-1} \delta(r-\mu). \quad (96)$$

For the accommodation strategy the value matching and smooth pasting conditions are given by

$$A_L^{acc} X^\beta = \frac{(X - \delta(r-\mu))^2}{8X\eta(r-\mu)}, \quad (97)$$

$$\beta A_L^{acc} X^{\beta-1} = \frac{X^2 - \delta^2(r-\mu)^2}{8X^2\eta(r-\mu)}. \quad (98)$$

Substitution of (98) into (97) gives

$$\frac{(X - \delta(r-\mu))^2}{8X\eta(r-\mu)} - \frac{X^2 - \delta^2(r-\mu)^2}{8\beta X\eta(r-\mu)} = 0, \quad (99)$$

from which we derive that

$$\frac{(\beta(X - \delta(r-\mu)) - (X + \delta(r-\mu)))(X - \delta(r-\mu))}{8\beta X\eta(r-\mu)} = 0. \quad (100)$$

Since from (97) it follows that $X = \delta(r-\mu)$ is not a valid solution, we have that

$$X_L^{acc} = \frac{\beta+1}{\beta-1} \delta(r-\mu). \quad (101)$$

Proof of Proposition 5 Given in the text.

Proof of Proposition 6 Given in the text.

Proof of Proposition 7 We know from the literature (e.g., Dixit and Pindyck (1994)) that

$$\frac{\partial \beta}{\partial \sigma} < 0. \quad (102)$$

Furthermore, we have that

$$\frac{\partial X_1^{acc}}{\partial \beta} = \frac{-4\delta(r-\mu)}{(\beta-1)^2} < 0, \quad (103)$$

$$\frac{\partial X_2^{det}}{\partial \beta} = \frac{-4\delta(r-\mu)}{(\beta-1)^2} < 0, \quad (104)$$

Concerning X_1^{det} it holds that

$$\psi(X_1^{det}, \beta) = \frac{X}{r - \mu} - \delta - \left(\frac{X(\beta - 1)}{(\beta + 1)\delta(r - \mu)} \right)^\beta \frac{\delta}{\beta - 1} = 0. \quad (105)$$

To see how X_1^{det} depends on β , we obtain from (105) that

$$\frac{\partial \psi(X, \beta)}{\partial X} \Big|_{X=X_1^{det}} \frac{\partial X_1^{det}}{\partial \beta} + \frac{\partial \psi(X, \beta)}{\partial \beta} \Big|_{X=X_1^{det}} = 0. \quad (106)$$

Rewriting gives

$$\frac{\partial X_1^{det}}{\partial \beta} = - \frac{\frac{\partial \psi(X, \beta)}{\partial \beta} \Big|_{X=X_1^{det}}}{\frac{\partial \psi(X, \beta)}{\partial X} \Big|_{X=X_1^{det}}}. \quad (107)$$

We know from (77) that $\frac{\partial \psi(X, \beta)}{\partial X} \Big|_{X=X_1^{det}} > 0$. Furthermore,

$$\frac{\partial \psi(X, \beta)}{\partial \beta} = - \frac{\delta}{\beta^2 - 1} \left(\frac{X(\beta - 1)}{(\beta + 1)\delta(r - \mu)} \right)^\beta \left(1 + (\beta + 1) \log \left(\frac{X(\beta - 1)}{(\beta + 1)\delta(r - \mu)} \right) \right), \quad (108)$$

so that

$$\frac{\partial \psi(X, \beta)}{\partial \beta} \Big|_{X=X_1^{det}} > 0, \quad (109)$$

if and only if

$$1 + (\beta + 1) \log \left(\frac{X_1^{det}(\beta - 1)}{(\beta + 1)\delta(r - \mu)} \right) < 0. \quad (110)$$

Define $\overline{X} = \frac{\beta}{\beta - 1} \delta(r - \mu)$, then $X_1^{det} < \overline{X}$, as it holds that

$$\overline{X} < \frac{\beta + 1}{\beta - 1} \delta(r - \mu), \quad (111)$$

$$\psi(X_1^{det}, \beta) = 0, \quad (112)$$

$$\frac{\partial \psi(X)}{\partial X} > 0 \text{ for } X \in \left(0, \frac{\beta + 1}{\beta - 1} \delta(r - \mu) \right), \quad (113)$$

and

$$\psi(\overline{X}, \beta) = \frac{\delta}{\beta - 1} \left(1 - \left(\frac{\beta}{\beta + 1} \right)^\beta \right) > 0. \quad (114)$$

Hence, (110) holds if

$$1 + (\beta + 1) \log \left(\frac{\overline{X}(\beta - 1)}{(\beta + 1)\delta(r - \mu)} \right) < 0. \quad (115)$$

Substitution of the definition of \overline{X} gives

$$1 + (\beta + 1) \log \left(\frac{\beta}{\beta + 1} \right) < 0. \quad (116)$$

Define the function $\gamma(\beta)$ as follows

$$\gamma(\beta) = 1 + (\beta + 1) \log \left(\frac{\beta}{\beta + 1} \right). \quad (117)$$

We have that

$$\gamma(1) = 1 + 2 \log \left(\frac{1}{2} \right) < 0, \quad (118)$$

$$\lim_{\beta \rightarrow \infty} \gamma(\beta) = 0, \quad (119)$$

$$\frac{\partial \gamma(\beta)}{\partial \beta} = \frac{1}{\beta} + \log \left(\frac{\beta}{\beta+1} \right) > 0. \quad (120)$$

The last equation holds since $\beta > 1$ and

$$\left. \frac{\partial \gamma(\beta)}{\partial \beta} \right|_{\beta=1} = 1 + \log \left(\frac{1}{2} \right) > 0, \quad (121)$$

$$\lim_{\beta \rightarrow \infty} \frac{\partial \gamma(\beta)}{\partial \beta} = 0, \quad (122)$$

$$\frac{\partial^2 \gamma(\beta)}{\partial \beta^2} = -\frac{1}{\beta^2 + \beta^2} < 0. \quad (123)$$

We conclude that $\left. \frac{\partial \psi(X, \beta)}{\partial \beta} \right|_{X=X_1^{det}} > 0$ and therefore $\frac{\partial X_1^{det}}{\partial \beta} < 0$.

B Monopolist that can invest twice

This appendix investigates the consequences for the investment policy if the monopolist has two (instead of one) investment opportunities. The first investment brings the capacity of the firm from 0 to Q_1 and the second investment from Q_1 to Q_2 . To rule out disinvestment we assume that $Q_2 > Q_1 > 0$. The model is solved backwards. This means that first for a given capacity level Q_1 the second investment is analyzed. After that the first investment is studied given the optimal investment behavior for the second investment.

Proposition 8 *Consider a monopolist that can invest twice in time. The optimal investment triggers X_1^* and X_2^* , and the corresponding optimal capacity levels Q_1^* and Q_2^* are implicitly given by the following equations:*

$$1 - \frac{\beta Q_1^* \eta}{(1 - \eta Q_1^*)} - 2 \left(\frac{\beta (1 - 2\eta Q_1^*)}{(\beta + 1)(1 - \eta Q_1^*)} \right)^\beta = 0, \quad (124)$$

$$X_1^*(Q_1) = \frac{\beta \delta (r - \mu)}{(\beta - 1)(1 - \eta Q_1)}, \quad (125)$$

$$X_2^*(Q_1) = \frac{(\beta + 1) \delta (r - \mu)}{(\beta - 1)(1 - 2\eta Q_1)}, \quad (126)$$

$$Q_2^*(Q_1) = \frac{1 + (\beta - 1) \eta Q_1}{(\beta + 1) \eta}. \quad (127)$$

Proof of Proposition 8 The value of the firm at the moment of the second investment when the capacity of the firm increases from Q_1 to Q_2 is equal to

$$V_2(X, Q_1, Q_2) = \frac{X Q_2 (1 - \eta Q_2)}{r - \mu} - \delta (Q_2 - Q_1). \quad (128)$$

Before the second investment the value is equal to

$$F_2(X, Q_1) = \frac{XQ_1(1-\eta Q_1)}{r-\mu} + A_2X^\beta. \quad (129)$$

Let us denote the trigger of the second investment by X_2^* . The value matching and smooth pasting conditions are given by

$$\frac{XQ_1(1-\eta Q_1)}{r-\mu} + A_2X^\beta = \frac{XQ_2(1-\eta Q_2)}{r-\mu} - \delta(Q_2 - Q_1), \quad (130)$$

$$\frac{Q_1(1-\eta Q_1)}{r-\mu} + \beta A_2X^{\beta-1} = \frac{Q_2(1-\eta Q_2)}{r-\mu}. \quad (131)$$

Solving these equations gives

$$X_2^* = \frac{\beta}{\beta-1} \frac{(r-\mu)\delta}{(1-\eta(Q_1+Q_2))}, \quad (132)$$

$$A_2 = \frac{(X_2^*)^{1-\beta} (Q_2 - Q_1) (1-\eta(Q_1+Q_2))}{\beta(r-\mu)}. \quad (133)$$

The optimal Q_2 is determined by solving

$$\max_{Q_2 > Q_1} \left[\frac{XQ_2(1-\eta Q_2)}{r-\mu} - \delta(Q_2 - Q_1) \right]. \quad (134)$$

The first order condition is given by

$$\frac{X(1-\eta Q_2)}{r-\mu} - \frac{XQ_2\eta}{r-\mu} - \delta = 0, \quad (135)$$

which gives

$$Q_2^*(X) = \frac{1}{2\eta} \left(1 - \frac{(r-\mu)\delta}{X} \right). \quad (136)$$

Solving the system of equations (132) and (136) leads to the equations (126) and (127).

The value of the firm at the moment of the first investment is equal to

$$V_1(X, Q_1) = \frac{XQ_1(1-\eta Q_1)}{r-\mu} - \delta Q_1 + A_2X^\beta. \quad (137)$$

Before the first investment the value is given by

$$F_1(X) = A_1X^\beta. \quad (138)$$

Value matching and smooth pasting results in the following equations:

$$A_1X^\beta = \frac{XQ_1(1-\eta Q_1)}{r-\mu} - \delta Q_1 + A_2X^\beta, \quad (139)$$

$$\beta A_1X^{\beta-1} = \frac{Q_1(1-\eta Q_1)}{r-\mu} + \beta A_2X^{\beta-1}, \quad (140)$$

which give

$$X_1^* = \frac{\beta}{\beta-1} \frac{(r-\mu)\delta}{(1-\eta Q_1)}, \quad (141)$$

$$A_1 = A_2 + \frac{(X_1^*)^{1-\beta} Q_1(1-\eta Q_1)}{\beta(r-\mu)}. \quad (142)$$

The optimal Q_1 can be determined by maximizing the value of the firm at the moment of the first investment:

$$\max_{Q_1 \geq 0} \left[\frac{X Q_1 (1 - \eta Q_1)}{r - \mu} - \delta Q_1 + A_2(Q_1) X^\beta \right]. \quad (143)$$

The first order condition is given by

$$\frac{X(1 - \eta Q_1)}{r - \mu} - \frac{X Q_1 \eta}{r - \mu} - \delta + \frac{\partial A_2(Q_1)}{\partial Q_1} X^\beta = 0. \quad (144)$$

Note that

$$\begin{aligned} A_2(Q_1) &= \frac{\delta(1 - 2\eta Q_1)(X_2^*(Q_1))^{-\beta}}{(\beta - 1)(\beta + 1)\eta} \\ &= \frac{\delta(1 - 2\eta Q_1)}{(\beta - 1)(\beta + 1)\eta} \left(\frac{\beta + 1}{\beta - 1} \frac{(r - \mu)\delta}{(1 - 2\eta Q_1)} \right)^{-\beta} \\ &= \frac{\delta}{(\beta - 1)(\beta + 1)\eta} \left(\frac{(\beta + 1)(r - \mu)\delta}{(\beta - 1)} \right)^{-\beta} (1 - 2\eta Q_1)^{\beta + 1}, \end{aligned} \quad (145)$$

so that

$$\begin{aligned} \frac{\partial A_2(Q_1)}{\partial Q_1} &= \frac{\delta}{(\beta - 1)(\beta + 1)\eta} \left(\frac{(\beta + 1)(r - \mu)\delta}{(\beta - 1)} \right)^{-\beta} (\beta + 1)(1 - 2\eta Q_1)^\beta \cdot -2\eta \\ &= -\frac{2\delta}{(\beta - 1)} \left(\frac{(\beta + 1)(r - \mu)\delta}{(\beta - 1)(1 - 2\eta Q_1)} \right)^{-\beta} \\ &= -\frac{2\delta(X_2^*(Q_1))^{-\beta}}{(\beta - 1)}. \end{aligned} \quad (146)$$

Substitution of (146) into (144) gives

$$\frac{X(1 - \eta Q_1)}{r - \mu} - \frac{X Q_1 \eta}{r - \mu} - \delta - \frac{2\delta}{(\beta - 1)} \left(\frac{X}{X_2^*(Q_1)} \right)^\beta = 0. \quad (147)$$

Substitution of (126) and (141) into equation (147) results in equation (124).

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